

HOW GOOD ARE SIMPLE RULES FOR ESTIMATING RIFLING TWIST

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Summary

Several simple rules for estimating twist, some including the stability factor, do not need the shape and mechanical properties of the bullet. Here I compare how well such rules fit actual experimental data for 14 bullets at various velocities (40 examples.) The new Miller Twist Rule, presented in *Precision Shooting*, March 2005, turns out to be significantly better than the others. The Harris Rule works very poorly.

Introduction

Most shooters know that a rifling twist that stabilizes a short bullet may not stabilize a longer one. The pressure to replace lead bullets with other metals, as in California, makes this increasingly important. Bullets using popular replacement metals, such as copper and bronze, are longer for the same weight than are lead-based ones. Therefore, many rifles may have too slow a twist to stabilize such non-lead bullets.

The obvious question is what twist do we need to get sufficient stability in such non-standard bullet and rifle combinations.

The required twist depends

mostly on the bullet's length, but also on its shape and velocity. Shape determines its mechanical properties, such as its weight, center of gravity, and the two moments of inertia. Shape also determines the overturning moment coefficient, an aerodynamic coefficient that depends on the velocity. Air density is also important (cold weather can make a barely-stable bullet unstable.) All these quantities appear in the mathematical formulas that describe stability.

Sufficient gyroscopic stability is necessary to keep a bullet flying point forward. However, with too much stability (overstability), poor quality bullets can give larger group sizes. These issues are characterized by the *stability factor*, whose value must be greater than 1.0.

The military usually chooses stability factors between 1.5 and 2.5. To protect against cold weather, 2.0 is safe, and 1.5 is suitable for most applications. Bench Resters often opt for 1.3 to minimize the effects of bullet imperfection, but that can be risky at low temperature or if the stability factor was set for high altitudes. However, stability factors even as high as 3.5 to 4.0 are usually not detri-

mental to small arms shooting, which is at low angles of elevation ("flat fire".)

Most twist rules have their origin in mathematical equations for stability, including Greenhill's original 1879 rule [G79,M05a,M06]. Miller's 2005 rule [M05,M08a] is directly based on the modern stability equations, and has the stability factor explicit, as do the Harris and Miller-Greenhill rules. See equations (1, 3, 6) on next page.

Although "fast design" computer programs exist for estimating the aerodynamic coefficients and mechanical properties needed for stability calculations, they require details about the construction and shape of the bullet. Unfortunately, these data are available for only a few of the several thousand sporting bullets. The simpler semi-empirical rules we test below ignore these shape factors, so should help in analyzing stability and twist for that large number of sporting bullets without such data.

Twist Rules To Be Tested

Notation: bullet diameter d in inches, rifling twist T in inches/turn or t in calibers/turn ($t=T/d$) (one turn is one complete rotation), bullet length L in inches or l in calibers ($l=L/d$), bullet weight m

in grains, velocity v in ft/sec, and stability factor s (dimensionless). I choose Army Standard Metro standard conditions, and f_v is a velocity correction. The air density correction f_a depends on temperature, pressure, or alternatively on altitude above sea level [M05, M08a]. The standard-conditions altitude correction is $f_a = \exp(3.158 \times 10^{-5} \times h)$ (h =altitude in feet) [McC99] and is important. If the twist is set for a specific stability factor at 7000 ft, but the gun is shot at sea level, the sea-level stability factor will be about 20% less! Conversely, for the same stability factor, the twist must be about 10% smaller.

We test the following simple new and old rules for estimating rifling twist, equations (1-7) below.

Equations (1) and (2) were first described in the March 2005 *Precision Shooting* [M05, M08a]. They were obtained from the modern stability equation [McC99], with correlations of moments of inertia and the overturning moment (and its velocity dependence) from experimental data in BRL Reports. By (blind stupid) good luck, the shape factors mostly cancel for *solid* and *solid-core* bullets. Below the velocity of sound, 1120 ft/sec, use the velocity correction f_v for 1120 ft/sec. An Excel program, by Bryan Litz and me, is available for calculating (a) twist given stability factor, (b) stability factor given twist, or (c) maximum length of bullet for a given twist and stability factor.

Equation (3) is less well known, and is due to C. E. Harris. I thank Ted Almgren of Sierra Bullets for its formula. Excel programs for it are on the Internet (search for Peter Cronhelm).

Equation (4) is the “classical” Greenhill formula found in the *British Textbook of Small Arms*,

(1) $T_M = d \sqrt{\frac{30m}{s d^3 l (1+l^2)}} f_v^{1/2} = \sqrt{\frac{30md^2}{s L (d^2 + L^2)}} f_v^{1/2}$ Miller with velocity (v) correction

where

(2) $f_v^{1/2} = \left(\frac{v}{2800}\right)^{1/6}$, $f_s = \left(\frac{v}{2800}\right)^{1/3}$ v corrections for twist (T) and stability factor (s), respectively

(3) $T_{MH} = \frac{20.62}{l^{2.25}} \sqrt{\frac{5705m}{s(5705-v)}} = \frac{1557.46}{l^{2.25}} \sqrt{\frac{m}{s(5705-v)}}$ Harris

(4) $T_{Gden} = \frac{150 d}{l} \sqrt{\frac{\text{density}}{10.9}}$ “Classical” Greenhill with density correction

(5) $T_{vGden} = \frac{150 d}{l} \sqrt{\frac{\text{density}}{10.9}} \left(\frac{v}{2800}\right)^{1/6}$ Greenhill with density and Miller v corrections

(6) $T_{MGden} = \frac{216.4 d}{l \sqrt{s}} \sqrt{\frac{\text{density}}{10.9}} \left(\frac{v}{2800}\right)^{1/6}$ Miller-Greenhill with s , density, and v corrections

(7) $T_{Gden \sqrt{v}} = \frac{3.5 d \sqrt{v}}{l} \sqrt{\frac{\text{density}}{10.9}}$ Various authors

| Cartridge | m gr. | d in. | base type | l cal. | s exp | v exp fps | T exp inches | bullet dens g/cm ³ | Source |
|-------------------------------------|--------|-------|-----------|--------|-------|-----------|--------------|-------------------------------|---------------|
| Powerlokt (.17 cal) | 26.2 | 0.172 | BT | 3.14 | 4.00 | 2800 | 6.00 | 10.00 | BRL 1630 p80 |
| Sierra 55 gr. | 55.5 | 0.224 | BT | 3.54 | 1.99 | 3024 | 12.00 | 10.00 | BRL-1630 p81 |
| M193 Ball | 55.0 | 0.224 | BT | 3.36 | 1.38 | 3080 | 12.00 | 7.30 | BRL-1630 p73 |
| Win. Western 190 Silvertip | 180.0 | 0.305 | M3 | 3.98 | 2.40 | 2464 | 10.00 | 10.00 | BRL-1630 p101 |
| Win. Western 190 Powerpoint | 179.0 | 0.305 | FR | 3.82 | 3.10 | 2464 | 10.00 | 10.00 | BRL-1630 p87 |
| .30 M73 Match | 173.0 | 0.305 | BT | 4.25 | 2.00 | 2640 | 10.00 | 10.00 | Harris AR |
| 4.32mm Ball (.17 cal) | 27.5 | 0.172 | BT | 3.69 | 1.48 | 3696 | 7.57 | 7.30 | BRL-1630 p99 |
| " | 27.5 | 0.172 | BT | 3.69 | 1.32 | 3046 | 7.57 | 7.30 | BRL-1630 |
| " | 27.5 | 0.172 | BT | 3.69 | 1.10 | 1926 | 7.57 | 7.30 | BRL-1630 |
| Federal 68 gr. | 66.0 | 0.224 | BT | 4.17 | 1.16 | 2688 | 10.00 | 10.00 | BRL-1630 p123 |
| " | 66.0 | 0.224 | BT | 4.17 | 1.07 | 2240 | 10.00 | 10.00 | BRL-1630 p123 |
| " | 66.0 | 0.224 | BT | 4.17 | 1.79 | 1568 | 7.50 | 10.00 | BRL-1630 p123 |
| " | 66.0 | 0.224 | BT | 4.17 | 1.73 | 1120 | 7.50 | 10.00 | BRL-1630 p123 |
| Sierra International Bullet 168 gr. | 166.1 | 0.305 | BT | 3.98 | 1.70 | 2478 | 12.00 | 9.64 | BRL-3733 |
| " | 166.1 | 0.305 | BT | 3.98 | 1.58 | 2017 | 12.00 | 9.64 | BRL-3733 |
| " | 166.1 | 0.305 | BT | 3.98 | 1.49 | 1602 | 12.00 | 9.64 | BRL-3733 |
| " | 166.1 | 0.305 | BT | 3.98 | 1.50 | 1266 | 12.00 | 9.64 | BRL-3733 |
| M118 Sierra 175 gr. MK | 172.9 | 0.305 | BT | 4.19 | 2.00 | 2442 | 10.00 | 9.82 | BRL-3733 |
| " | 172.9 | 0.305 | BT | 4.19 | 1.86 | 2039 | 10.00 | 9.82 | BRL-3733 |
| " | 172.9 | 0.305 | BT | 4.19 | 1.72 | 1568 | 10.00 | 9.82 | BRL-3733 |
| " | 172.9 | 0.305 | BT | 4.19 | 1.60 | 1221 | 10.00 | 9.82 | BRL-3733 |
| Sierra 190 gr. MK | 189.4 | 0.305 | BT | 4.31 | 1.94 | 2431 | 10.00 | 10.34 | BRL-3733 |
| " | 189.4 | 0.305 | BT | 4.31 | 1.82 | 2028 | 10.00 | 10.34 | BRL-3733 |
| " | 189.4 | 0.305 | BT | 4.31 | 1.70 | 1557 | 10.00 | 10.34 | BRL-3733 |
| " | 189.4 | 0.305 | BT | 4.31 | 1.62 | 1266 | 10.00 | 10.34 | BRL-3733 |
| .30 M33 Ball | 646.5 | 0.510 | BT | 4.46 | 1.81 | 2932 | 15.00 | 7.49 | BRL-MR-3820 |
| " | 646.5 | 0.510 | BT | 4.46 | 1.61 | 2190 | 15.00 | 7.49 | BRL-MR-3820 |
| " | 646.5 | 0.510 | BT | 4.46 | 1.48 | 1680 | 15.00 | 7.49 | BRL-MR-3820 |
| " | 646.5 | 0.510 | BT | 4.46 | 1.43 | 1333 | 15.00 | 7.49 | BRL-MR-3820 |
| " | 646.5 | 0.510 | BT | 4.46 | 1.33 | 1143 | 15.00 | 7.49 | BRL-MR-3820 |
| BRL 1 Cu plated steel | 66.0 | 0.224 | BT | 5.48 | 1.52 | 3136 | 6.00 | 7.30 | BRL-1630 p173 |
| " | 66.0 | 0.224 | BT | 5.48 | 1.46 | 2688 | 6.00 | 7.30 | BRL-1630 |
| " | 66.0 | 0.224 | BT | 5.48 | 1.28 | 1904 | 6.00 | 7.30 | BRL-1630 |
| " | 66.0 | 0.224 | BT | 5.48 | 1.18 | 1344 | 6.00 | 7.30 | BRL-1630 |
| " | 66.0 | 0.224 | BT | 5.48 | 1.16 | 1277 | 6.00 | 7.30 | BRL-1630 |
| ANSX-S solid Bural | 1037.0 | 0.727 | FB | 5.00 | 2.67 | 2868 | 11.00 | 2.80 | BRL-876 |
| " | 1037.0 | 0.727 | FB | 5.00 | 2.60 | 2736 | 11.00 | 2.80 | BRL-876 |
| " | 1037.0 | 0.727 | FB | 5.00 | 2.49 | 2717 | 11.00 | 2.80 | BRL-876 |
| " | 1037.0 | 0.727 | FB | 5.00 | 2.54 | 2484 | 11.00 | 2.80 | BRL-876 |
| " | 1037.0 | 0.727 | FR | 5.00 | 1.85 | 2034 | 12.00 | 2.80 | BRL-876 |

How Good Are Simple Rules For Estimating Rifling Twist *Continued*

1929 [TSA29], with the correction for bullet *density* (in gram/cm³) made explicit. This formula has no velocity correction because it was based on subsonic flow. It is a good approximation to Greenhill's much-more-complicated original formula [M05a,M06]. At 2800 ft/sec (Mach number M=2.5), it corresponds to stability factors of 1.5-2.0 [M05a,M06], as also found by C. E. Harris and Bill Davis.

Equation (5) is equation (4) with the velocity correction of equation (2).

Equation (6) looks much like the Greenhill formula, but it includes the stability factor *s* and the velocity

correction of equation (2). It is considerably better and was independently derived from a variant of equation (1) with a further assumption about bullet volume [M08c].

Equation (7) has a number of similar versions from various authors with coefficients other than 3.5, all of which use a velocity correction factor of $v^{1/2}$. I have added the explicit bullet-density correction. I have no original source for equation (7), so I apologize to its original author.

Sources of Input Data

These twist rules were tested on 40 examples for 14 bullets from various sources, almost all from Ballistic Research Lab reports. Such experimentally-measured data, which include weight, twist, stability factor, bullet length, and

velocity, are *very* hard to find. These 40 examples include all the sporting bullets plus some military ball or solid test projectiles that are readily available from the military literature. These more or less fit into the solid or solid-core condition for the Miller Rule [M05,M08a]. The raw data for the .308 Sierra and .50 M33 bullets were corrected to zero yaw by standard methods [McC99]. Their *s* data were clustered at their corresponding clustered Mach number values, and both were averaged [M08a]. The 40 examples include 4 data sets at velocities other than those presented in my *Precision Shooting Part I* article [M08a]. These are the three .308 Sierra and the .224 Federal bullets. **Table I** contains the input data. (exp means experimental values.)

Greenhill-type rules require bullet densities. Some bullets were made of known materials: the 4.32 mm ball (a steel test bullet), BRL-1 (Cu plated steel), and ANSR-5 (Dural aluminum). Some could be estimated from their volume (calculated from their known shapes) and weight (.308 Sierra 168, 175, and 190 grain, and the .50 M33.) The other estimates are somewhat uncertain. There are also some uncertainties in the description of the guns and twist for the .224 Federal 68 grain in BRL-1630, where only one twist was stated. The other had to be estimated from $T=2\pi d/spin$.

Tests

The test for each example consists of estimating the twist from the various equations (1-7) using measured values of bullet weight, diameter, and length, plus the *stability factor* and density when necessary. This estimate was compared with the measured twist for the test gun. **Table II** contains the results.

Table III contains the residuals (errors), where *residuals* are defined as (estimated — experimental).

Table II. Estimated Twists (inches/turn)

| Cartridge | v exp fps | T exp | Tv Miller | Tv Harris | T Gden | Tv Gden | Tv MGden | Tsq(v) Gden |
|--|--------------|-------|--------------|--------------|-----------|------------|-------------|----------------|
| Powerlokt (.17 cal) | 2800 | 6.00 | 5.79 | 5.63 | 7.87 | 7.87 | 5.68 | 9.72 |
| Sierra 55 gr. | 3024 | 12.00 | 11.11 | 11.48 | 9.09 | 9.21 | 11.70 | 11.67 |
| M193 Ball | 3080 | 12.00 | 11.66 | 12.67 | 8.51 | 8.65 | 10.62 | 11.02 |
| Win. Western 180 Silvertip | 2464 | 10.00 | 10.45 | 10.96 | 11.29 | 11.05 | 10.29 | 13.07 |
| Win. Western 180 Powerpoint | 2464 | 10.00 | 10.27 | 11.50 | 12.22 | 11.97 | 9.81 | 14.16 |
| .30 M72 Match | 2640 | 10.00 | 10.10 | 10.09 | 10.41 | 10.31 | 10.52 | 12.48 |
| 4.32mm Ball (.17 cal) | 3696 | 7.87 | 7.53 | 7.05 | 5.65 | 5.91 | 7.01 | 8.01 |
| " | 3046 | 7.87 | 7.72 | 6.49 | 5.65 | 5.73 | 7.19 | 7.27 |
| " | 1926 | 7.87 | 7.83 | 5.96 | 5.65 | 5.31 | 7.30 | 5.78 |
| Federal 68 gr. | 2688 | 10.00 | 10.05 | 8.74 | 7.72 | 7.67 | 10.27 | 9.34 |
| " | 2240 | 10.00 | 10.15 | 8.49 | 7.72 | 7.44 | 10.37 | 8.52 |
| " | 1568 | 7.50 | 7.40 | 6.01 | 7.72 | 7.01 | 7.56 | 7.13 |
| " | 1120 | 7.50 | 7.11 | 5.80 | 7.72 | 6.62 | 7.27 | 6.03 |
| Sierra International Bullet 168 gr. | 2478 | 12.00 | 11.75 | 12.19 | 10.92 | 10.70 | 11.84 | 12.68 |
| " | 2017 | 12.00 | 11.77 | 11.82 | 10.92 | 10.34 | 11.86 | 11.44 |
| " | 1602 | 12.00 | 11.67 | 11.54 | 10.92 | 9.95 | 11.76 | 10.20 |
| " | 1266 | 12.00 | 11.18 | 11.06 | 10.92 | 9.56 | 11.27 | 9.06 |
| M118 Sierra 175 gr. MK | 2442 | 10.00 | 10.20 | 10.12 | 10.47 | 10.23 | 10.44 | 12.07 |
| " | 2039 | 10.00 | 10.27 | 9.90 | 10.47 | 9.93 | 10.50 | 11.03 |
| " | 1568 | 10.00 | 10.22 | 9.69 | 10.47 | 9.50 | 10.45 | 9.67 |
| " | 1221 | 10.00 | 10.16 | 9.65 | 10.47 | 9.11 | 10.39 | 8.53 |
| Sierra 190gr. MK | 2431 | 10.00 | 10.37 | 10.05 | 10.44 | 10.20 | 10.56 | 12.01 |
| " | 2028 | 10.00 | 10.36 | 9.76 | 10.44 | 9.89 | 10.55 | 10.97 |
| " | 1557 | 10.00 | 10.28 | 9.54 | 10.44 | 9.47 | 10.48 | 9.61 |
| " | 1266 | 10.00 | 10.18 | 9.44 | 10.44 | 9.15 | 10.37 | 8.67 |
| .50 M33 Ball | 2913 | 15.00 | 15.14 | 19.30 | 14.22 | 14.31 | 15.35 | 17.91 |
| " | 2190 | 15.00 | 15.31 | 18.24 | 14.22 | 13.65 | 15.52 | 15.53 |
| " | 1680 | 15.00 | 15.27 | 17.78 | 14.22 | 13.06 | 15.49 | 13.60 |
| " | 1333 | 15.00 | 14.95 | 17.35 | 14.22 | 12.56 | 15.16 | 12.11 |
| " | 1143 | 15.00 | 15.11 | 17.61 | 14.22 | 12.25 | 15.32 | 11.22 |
| BRL-1 Cu plated steel | 3136 | 6.00 | 5.96 | 4.41 | 5.22 | 5.32 | 6.22 | 6.82 |
| " | 2688 | 6.00 | 5.93 | 4.15 | 5.22 | 5.18 | 6.19 | 6.31 |
| " | 1904 | 6.00 | 5.98 | 3.95 | 5.22 | 4.89 | 6.24 | 5.31 |
| " | 1344 | 6.00 | 5.87 | 3.84 | 5.22 | 4.62 | 6.13 | 4.47 |
| " | 1277 | 6.00 | 5.87 | 3.84 | 5.22 | 4.58 | 6.13 | 4.35 |
| ANSR-5 solid Dural | 2868 | 11.00 | 10.71 | 15.41 | 11.97 | 12.01 | 10.61 | 14.95 |
| " | 2736 | 11.00 | 10.77 | 15.27 | 11.97 | 11.92 | 10.67 | 14.60 |
| " | 2717 | 11.00 | 11.00 | 15.55 | 11.97 | 11.91 | 10.89 | 14.55 |
| " | 2484 | 11.00 | 10.73 | 14.83 | 11.97 | 11.73 | 10.62 | 13.92 |
| " | 2034 | 12.00 | 12.16 | 16.28 | 11.97 | 11.35 | 12.03 | 12.59 |

Therefore, a negative residual represents a smaller (faster, quicker, shorter) estimated twist in inches per turn, and a positive one means a larger (slower) twist. **Table IV** is a summary of statistical quantities for the residuals and percent residuals, where the percent residual (not tabulated here) is the residual divided by the experimental twist. When we reverse calculate the stability factor *s* from twist, the percentage error doubles.

Note that equation (1) was originally obtained [M05] using data that included some of the bullets here but also some 20 others as well, all at 2800 ft/sec. The velocity correction, equation (2), was then obtained from data for other velocities. Therefore, the test here is independent.

The calculated results are given to two decimals for comparisons, although the uncertainties in the estimates are larger than one decimal.

Results

Tables III and IV show the following.

The Miller Rule, equation (1), is distinctly the best, with a residual standard deviation of only 0.3". Furthermore, 95% of the estimates are within 0.5" (compatible with the statistical 2 standard deviations). Its percentage residuals are equally encouraging, with a standard deviation of 2.8%. Furthermore, 95% are within 5.2% (again compatible with statistics.) The maximum deviation from experiment is -0.9", and fortunately a minus residual means the twist is more conservatively estimated, i.e., is faster and therefore safer against cold weather or lower altitudes. The worst examples are the .224 Sierra 55 grain and the lowest-velocity Sierra International Bullet.

Table III. Twist Residuals (inches/turn)

| Cartridge | v exp fps | res Tv M | res Tv H | res T Gden | res Tv Gden | res Tv MG den | res Tsq (v) Gden |
|--|--------------|-------------|-------------|---------------|----------------|------------------|------------------------|
| Powerlokt (.17 cal) | 2800 | -0.21 | -0.37 | 1.87 | 1.87 | -0.32 | 3.72 |
| Sierra 55 gr. | 3024 | -0.89 | -0.52 | -2.91 | -2.79 | -0.30 | -0.33 |
| M193 Ball | 3080 | -0.34 | 0.67 | -3.49 | -3.35 | -1.38 | -0.98 |
| Win. Western 180 Silvertip | 2464 | 0.45 | 0.96 | 1.29 | 1.05 | 0.29 | 3.07 |
| Win. Western 180 Powerpoint | 2464 | 0.27 | 1.50 | 2.22 | 1.97 | -0.19 | 4.16 |
| .30 M72 Match | 2640 | 0.10 | 0.09 | 0.41 | 0.31 | 0.52 | 2.48 |
| 4.32mm Ball (.17 cal) | 3696 | -0.34 | -0.82 | -2.22 | -1.96 | -0.86 | 0.14 |
| " | 3046 | -0.15 | -1.38 | -2.22 | -2.14 | -0.68 | -0.60 |
| " | 1926 | -0.04 | -1.91 | -2.22 | -2.56 | -0.57 | -2.09 |
| Federal 68 gr. | 2688 | 0.05 | -1.26 | -2.28 | -2.33 | 0.27 | -0.66 |
| " | 2240 | 0.15 | -1.51 | -2.28 | -2.56 | 0.37 | -1.48 |
| " | 1568 | -0.10 | -1.49 | 0.22 | -0.49 | 0.06 | -0.37 |
| " | 1120 | -0.39 | -1.70 | 0.22 | -0.88 | -0.23 | -1.47 |
| Sierra International Bullet 168 gr. | 2478 | -0.25 | 0.19 | -1.08 | -1.30 | -0.16 | 0.68 |
| " | 2017 | -0.23 | -0.18 | -1.08 | -1.66 | -0.14 | -0.56 |
| " | 1602 | -0.33 | -0.46 | -1.08 | -2.05 | -0.24 | -1.80 |
| " | 1266 | -0.82 | -0.94 | -1.08 | -2.44 | -0.73 | -2.94 |
| M118 Sierra 175 gr.MK | 2442 | 0.20 | 0.12 | 0.47 | 0.23 | 0.44 | 2.07 |
| " | 2039 | 0.27 | -0.10 | 0.47 | -0.07 | 0.50 | 1.03 |
| " | 1568 | 0.22 | -0.31 | 0.47 | -0.50 | 0.45 | -0.33 |
| " | 1221 | 0.16 | -0.35 | 0.47 | -0.89 | 0.39 | -1.47 |
| Sierra 190 gr.MK | 2431 | 0.37 | 0.05 | 0.44 | 0.20 | 0.56 | 2.01 |
| " | 2028 | 0.36 | -0.24 | 0.44 | -0.11 | 0.55 | 0.97 |
| " | 1557 | 0.28 | -0.46 | 0.44 | -0.53 | 0.48 | -0.39 |
| " | 1266 | 0.18 | -0.56 | 0.44 | -0.85 | 0.37 | -1.33 |
| .50 M33 Ball | 2913 | 0.14 | 4.30 | -0.78 | -0.69 | 0.35 | 2.91 |
| " | 2190 | 0.31 | 3.24 | -0.78 | -1.35 | 0.52 | 0.53 |
| " | 1680 | 0.27 | 2.78 | -0.78 | -1.94 | 0.49 | -1.40 |
| " | 1333 | -0.05 | 2.35 | -0.78 | -2.44 | 0.16 | -2.89 |
| " | 1143 | 0.11 | 2.61 | -0.78 | -2.75 | 0.32 | -3.78 |
| BRL-1 Cu plated steel | 3136 | -0.04 | -1.59 | -0.78 | -0.68 | 0.22 | 0.82 |
| " | 2688 | -0.07 | -1.85 | -0.78 | -0.82 | 0.19 | 0.31 |
| " | 1904 | -0.02 | -2.05 | -0.78 | -1.11 | 0.24 | -0.69 |
| " | 1344 | -0.13 | -2.16 | -0.78 | -1.38 | 0.13 | -1.53 |
| " | 1277 | -0.13 | -2.16 | -0.78 | -1.42 | 0.13 | -1.65 |
| ANSR-5 solid Dural | 2868 | -0.29 | 4.41 | 0.97 | 1.01 | -0.39 | 3.95 |
| " | 2736 | -0.23 | 4.27 | 0.97 | 0.92 | -0.33 | 3.60 |
| " | 2717 | -0.00 | 4.55 | 0.97 | 0.91 | -0.11 | 3.55 |
| " | 2484 | -0.27 | 3.83 | 0.97 | 0.73 | -0.38 | 2.92 |
| " | 2034 | 0.16 | 4.28 | -0.03 | -0.65 | 0.03 | 0.59 |

Table IV. Summary of Residual and Percentage Residual Statistics

| | | Twist Residuals in inches/turn | | | | | |
|---------|--------|--------------------------------|------------------|---------------|----------------|------------------|------------------------|
| | | res Tv Miller | res Tv Harris | res T Gden | res Tv Gden | res Tv MG den | res Tsq (v) Gden |
| StDev | inches | 0.30 | 2.12 | 1.30 | 1.34 | 0.45 | 2.10 |
| Minimum | inches | -0.89 | -2.16 | -3.49 | -3.35 | -1.38 | -3.78 |
| Maximum | inches | 0.45 | 4.55 | 2.22 | 1.97 | 0.56 | 4.16 |
| Range | inches | 1.34 | 6.72 | 5.71 | 5.32 | 1.94 | 7.94 |

| | | Percentage Twist Residuals | | | | | |
|---------|---------|----------------------------|--------------------|-----------------|------------------|--------------------|--------------------------|
| | | % res Tv Miller | % res Tv Harris | % res T Gden | % res Tv Gden | % res Tv MG den | % res Tsq (v) Gden |
| StDev | percent | 2.8 | 21.4 | 14.0 | 14.0 | 4.6 | 21.5 |
| Minimum | percent | -7.4 | -36.0 | -29.1 | -32.6 | -11.5 | -27.5 |
| Maximum | percent | 4.5 | 41.4 | 31.2 | 31.2 | 5.6 | 62.0 |
| Range | percent | 11.9 | 77.4 | 60.2 | 63.8 | 17.1 | 89.4 |

Range is the maximum spread, i.e., the maximum minus the minimum.

How Good Are Simple Rules For Estimating Rifling Twist *Continued*

The Miller-Greenhill formula, equation (6), is next best and surprisingly good. The standard deviation and the range (maximum spread) of both the residuals and percentage residuals are about 50% larger than for the Miller rule, i.e., 0.45" and 5.4%, with 95% of these two types of residuals within about 3/4" and 9% respectively. The worst case is the M193 Ball.

The two classical Greenhill cases, equation (4) without velocity correction and equation (5) with the Miller velocity correction, are next best and close to each other. However, their standard deviations are considerably larger, and about 4½ to 5 times larger than the Miller Rule's.

The Harris Rule, equation (3), is even worse and surprisingly poor. A residual plot (not shown) shows a steady increase with d : with negative values at low d , fairly good results for d from about .270 to .338, and positive values above that. The residual standard deviation is 7 times larger and the percentage residual standard deviation is about 5 times larger than the corresponding Miller ones, and about double the ones for the Miller-Greenhill case. Although the d dependence can be reduced by an empirical power of d in the denominator, its errors are still double those of the Miller-Greenhill Rule, so that correction does not seem useful. Sierra Bullets uses Harris' Rule for comparisons not designs, and bases its twist recommendations on long experience.

Finally, the worst is the Greenhill with the $v^{1/2}$ correction factor, equation (7), which was expected to be poor. Its residual standard deviation is about the same as Harris', and the percentage one is about 10% larger than Harris'.

Conclusions

The analysis above suggests that the best estimation rule for rifling twist (and in reverse the stability factor) is the Miller Rule, equation (1). This is naturally very gratifying to me. The next best is the Miller-Greenhill Rule. Both are easy to use. However, the Miller Rule not only gives better results, but uses readily-available bullet weights in contrast to Miller-Greenhill, which uses the less-available bullet densities.

The Miller Rule is recommended, because so far it's both the best simple rule for estimating rifling twists or stability factors of sporting bullets, and therefore a tool for exploring non-standard bullet-rifle combinations.

*Finally, while the Miller Rule agrees very well with experiment here, such a semi-empirical rule cannot cover **all** cases.* There is anecdotal evidence that (1)

plastic-tip or hollow-base bullets may have larger errors, and (2) the velocity correction above 3600 ft/sec may predict faster twists than needed. Improvement must await new twist-stability factor data.

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