A New Rule for Estimating Rifling Twist
An Aid to Choosing Bullets and Rifles

by Don Miller ©

I. Introduction

For a bullet to fly point-forward, it must spin fast enough to be stable. For stability, the "gyroscopic stability factor" *s,* which depends on the spin rate, must be greater than 1.0. Because the spin in flight declines considerably more slowly than the forward velocity, the bullet's stability factor increases as it flies downrange, and it actually becomes more stable. Consequently, the stability factor at the muzzle is the most significant. Since the spin rate at the muzzle depends on rifling twist, twist is an important part of the stability factor.

It is well known that longer bullets need a faster rifling twist to be stable. This concerns shooters wanting to choose bullets for their rifles or to choose a rifle to use particular bullets. Therefore, such concerns raise many interesting questions. What level of stability is "safe"? What twist is enough for a safe stability? If a twist has a certain stability factor *s*, what is the stability factor for a different twist? What about too fast a twist (overstability)? What twist do I need for a long (heavy) bullet? If my rifle's twist is OK for a standard jacketed bullet, will it be OK for a bronze bullet of the same weight? How is the stability affected by muzzle velocity *v*, atmospheric air temperature *T* and pressure *P*, or altitude *h*?

The accurate answers to these questions require experimental data available only for a small number of military bullets and shells, but simply not available for the hundreds of sporting bullets.

What do we do in this situation? We use semi-empirical correlations. They aren't perfect, but can be quite good guides and get us into the ballpark.

Greenhill's old, simple rule is $t\,l = 150$, where *t* is the twist in calibers/turn and *l* is bullet length in calibers. Derived in 1879 for a football-shaped bullet at subsonic velocities **[M04],** it estimates a "safe" twist in terms of bullet length alone and assumes a density of 10.9 g/cm^3. It works much better than expected at modern velocities, with actual stability factors of 1.5-2.0 at 2800 ft/sec. However, it is not as good for Black Powder velocities.

There are also modern "fast design" programs that either estimate twist directly or are based on estimating twist from the "overturning moment" $C_{M\alpha}$. Public domain examples are Bob McCoy's McGyro and Intlift. However, besides the length, they typically require detailed and hard-to-get knowledge of bullet shape, moments of inertia, and the center of gravity. Unfortunately, they give conflicting results, and the conflicts are worse below about 1500 ft/sec.

While any discussion of twist rules necessarily requires formulas, using them just needs plugging numbers into them, as we will see in examples.

II. The New Rule for Twist and Stability Factor

I propose a new semi-empirical rule for estimating the necessary twist for a safe stability factor at **standard conditions** (Army Standard Metro) and Mach number **M=2.5** (a velocity of **2800**

ft/sec), or conversely, estimating the stability factor from the twist. This rule doe not depend on shape but includes one more parameter than Greenhill's Rule, the easily obtained weight of the bullet m. It is based on correlating experimental bullet data obtained at the US Army Research Laboratory (formerly Ballistic Research Laboratory). I include approximate corrections for velocity (Mach number), as well as adjustments for different air temperatures and barometric pressures.

This simple new rule is better and more general than the Greenhill formula. If m is the bullet weight in grains, s the gyroscopic stability factor (dimensionless), d the bullet diameter in inches, l the bullet length in calibers, and t the twist in calibers per turn, then our new twist rule for the *square* of the twist is

$$t^2 = \frac{30\,m}{s\,d^3\,l\,(1+l^2)}$$ (A)

where $t=T/d$ (T is the twist in inches per turn) and $l=L/d$ (L is the bullet length in inches.)

To get the stability factor s for a known twist, bullet weight, and length, eq A becomes

$$s = \frac{30\,m}{t^2\,d^3\,l\,(1+l^2)}$$ ` (B)

Note that the stability factor is inversely proportional to the **square** of the twist. Therefore, the twist itself is just the square root of eq A.

$$t = \sqrt{\frac{30\,m}{s\,d^3\,l\,(1+l^2)}} \qquad T = \sqrt{\frac{30\,m}{s\,d\,l\,(1+l^2)}}$$ (C)

For a given bullet and gun, eq A or B relates a different twist t_2 or stability factor s_2 to the original ones t_1 and s_1 by

$$s_2\,t_2^2 = s_1\,t_1^2$$ (D)

Eq A-C have the bullet length in the *denominator*. Therefore, a longer bullet means a lower stability factor or smaller (faster, tighter) twist. Note that m is in the *numerator*. For a given bullet length, boattail or hollow point bullets are lighter, as are bronze bullets. Therefore, they are less stable or need a tighter twist.

Our rule for t^2 (eq A) implicitly contains the bullet's density in the bullet weight term, so applies to cast lead bullets, jacketed bullets, bronze bullets, other solid core bullets, etc. without modification. However, the constant 30 only applies at M=2.5 (2800 ft/sec) and standard temperature and pressure conditions (59 degrees Fahrenheit, 750 mm Hg, and 78% humidity.)

If we want a safe twist for a bullet of a given length and weight, what is a safe gyroscopic stability factor that takes into account non-standard atmospheric conditions and velocities?

Recommendations for safe stability factors at *standard* conditions run from 1.3 to 2.0 **[H62,H65,H83,D88]**. Military practice is 1.5 to 2.0 **[McC99]**. These values help compensate for dynamic instability, which increases the minimum required gyroscopic stability factor above 1.0

[McC99]. W. C. Davis, Jr. **[D88]** stated that in his experience larger stability factors, even up to 3.5, don't seem to have bad effects on accuracy, and that "overstability is a myth." Therefore, let's start with s=1.75. However, low temperatures, like Duluth in the winter, significantly increase air densities and thus decrease s, because air density is implicit in the denominators of eq A-C.

   To automatically account for low temperatures, *we recommend using s=2.0 as the "safe s"* *for preliminary calculations of twist.*

   What about velocity effects on t and s? The overturning moment, implicit in the denominators of eq A-C, is the only velocity-dependent term. A **VERY** crude approximation to correct for its velocity dependence is to multiply the calculated *s* by the **1/3** root of (v/2800) and calculated *t* by the **1/6** root of (v/2800), where v is the velocity in ft/sec. Below the velocity of sound (1120 ft/sec, M=1.0), we use the velocity of sound value. This correction means stability factors or twists are smaller below 2800 ft/sec and higher above. This calculation requires a hand calculator with a y^x key. If we write a correction factor f_v as

$$f_v = (v/2800)^{1/3} \tag{E}$$

then

$$s_v = s_{2800}\, f_v = s_{2800}\,(v/2800)^{1/3} \qquad \text{(below 1120 ft/sec, use value for 1120)} \tag{F}$$

$$t_v = t_{2800}\, \sqrt{f_v} = t_{2800}\,(v/2800)^{1/6} \qquad \text{(below 1120 ft/sec, use value for 1120)} \tag{G}$$

   Corrections for temperature and pressure are based on the Perfect Gas Law, and can be combined into a factor f_{TP}, where

$$f_{TP} = \frac{\rho_{std}}{\rho_T} = \frac{(FT+460)}{(59+460)} \frac{P_{std}}{P_T} = \frac{(CT+273)}{(15+273)} \frac{P_{std}}{P_T} \tag{H}$$

where ρ_{std} is the air density at standard conditions, FT the measured Fahrenheit temperature, CT the measured Celsius (centigrade) temperature, and P_T the measured atmospheric pressure. This correction **multiplies** the calculated s and t^2, and its square root multiplies t itself.

   Finally there is a general estimate of the effect of altitude on air density, when actual measurements of T and P are not available. If h is the altitude in ft, then the correction factor f_h that **multiplies** s and t^2 is

$$f_h = \frac{\rho_{std}}{\rho_T} = e^{\left(+3.158\times10^{-5}\times h\right)} \tag{I}$$

   These correction factors f_v, f_{TP}, and f_h don't change t when calculating s using eq A, and don't change s when calculating t using eq B or C.

   These results are obtained by the analysis in Section IV.

   First, let's turn to some tests and examples of these formulas.

III. Sample Calculations

The following examples show how well the Rule works and how to answer the questions about twist and stability raised in the Introduction.

A. Comparison with Experiment at 2800 ft/sec (M=2.5)

Case 1. What is the *twist* for a given stability factor for the 168 gr. Sierra International Bullet **[McC88,99]**? For this case, m=168 gr., d=0.308", l=3.98 calibers, and the measured s=1.80 at 2800 ft/sec. Then putting these numbers into eq A, we get $t^2 = 30\,m/[s\,d^3\,l(1+l^2)] = 30\times 168/$ $[1.80\times 0.308^3\times 3.98\times(1+3.98^2)] = 1429.8$. Therefore $t = \sqrt{1429.8} = 37.813$ calibers per turn, and the calculated twist in inches per turn is $T=td$=37.813$\times$0.308=11.65". The actual twist was 12" per turn, for an error of only 3%.

Case 2. What is the *twist* for a given stability factor for the 5 caliber Army Navy Spinner Rocket made of Dural **[B73]**? Dural is an aluminum alloy, whose density is 2.8 compared to 10.9 for jacketed bullets. For this case, after converting to our units, m=1037 gr., d=0.7874", l=5.0 calibers, and the measured s=2.59. The calculated t^2=30$\times$1037/[2.59$\times$0.7874$^3\times$5$\times$(1+5^2)] = 189.27, so $t = \sqrt{189.27} = 13.76$ calibers per turn. The actual twist was 14 calibers, for an error of only 2%.

B. Analyzing the Dunham's Bay Case.

In a benchrest match at Dunham's Bay, NY in February 1988, Dr. Richard Maretzo had serious yawing (elongated holes) and unexpectedly wild groups with his 70 gr., 6 mm boat tail bullets using a gun with a 14"twist. The weather was very cold, -10 degrees Fahrenheit. Shorter, 70 gr. flat based bullets were not affected. Several letters about this incident in Precision Shooting (July, 1988) agreed that the increased air density from the cold weather had reduced the stability factor to the point of instability. The issue was beautifully analyzed in detail by ballistician Bill Davis, Jr. **[D88]**, who had gotten a 70 gr., 6 mm bullet and measured its dimensions on a comparator. He then used a computer program based on one by Bob McCoy of BRL, and showed that the bullet was indeed unstable at −10 degrees F at the muzzle velocity of 3350 ft/sec. He didn't give the length of the flat base bullets he used for comparison, so we can't compare our Rule for that case.

From Bill Davis' measurements, m=70 gr., d=0.243", bullet length was 0.83" or l=0.83/0.243=3.4156 calibers.

What do our formulas tell us about this case, considering the various questions raised in the Introduction.

First, *what is a safe twist at **2800** ft/sec and standard conditions*? Substituting our safe stability factor of s=2 and the bullet quantities into eq A gives t=41.13 calibers and T=41.13$\times$0.243=9.994", or a 10" twist.

Second, *what is the safe twist at **3350** ft/sec and standard conditions*, using our safe stability factor of s=2? For this we calculate f_v from eq E; f_v=(3350/2800)$^{1/3}$ = 1.1964$^{1/3}$ = 1.0616. Then T_{3350}=$T_{2800}f_v^{1/2}$=9.994$\times$1.0616$^{1/2}$= 9.994$\times$1.030=10.30" at 3350 ft/sec and standard conditions. This

is still about a 10" twist, and shows that increasing velocity doesn't make all that much increase in stability.

Third, *what is the stability factor in a 14" twist barrel at 3350 ft/sec and standard conditions?* From eq D, $s_{14}=s_{safe}\times(10.30/14)^2=2\times0.7356^2=2\times0.5413=1.083$. (Bill Davis got 1.16.) This stability factor is too low to be really safe even at 59 degrees F.

Fourth, *what is the stability factor in a 14" twist barrel at 3350 ft/sec, standard pressure, but at -10 F?* For this, we calculate f_{TP} from eq H: $f_{TP}=(460-10)/(519)=0.8671$. Then $s_{14(-10)}=1.083\times0.8671=0.939$. (Davis got 0.999.) But this is less than 1.0, so the bullet is **unstable** at -10 F with a 14" twist!

Fifth, *would this bullet be stable in a 13" twist at -10 F?* We again calculate the new *s* from eq D. $s_{13(-10)}=s_{14(-10)}(14/13)^2=0.939\times1.160=1.089$. Yes, the bullet is stable, but barely so at -10 F. (Davis got 1.16.)

Sixth, *how about a 12" twist at -10 F?* Again using eq D, we have $s_{12(-10)}=s_{13(-10)}\times(13/12)^2=1.089\times1.174=1.278$. This is at the very low end of design stability at -10 F. (Davis got 1.36.)

Finally, *what are the stability factors at standard conditions and 3350 ft/sec for the 12" and 13" twists.* Eq D gives $s_{12(59)}=1.473$ and $s_{13(59)}=1.256$. (Davis got 1.58 and 1.34, respectively.) The 12" twist with $s=1.473$ is at the low end of the usual design considerations, and the 13" twist with $s=1.256$ is borderline.

The upshot is that our calculations are close to Bill Davis' more elaborate ones, but consistently about 7% lower than his. Therefore, our stability estimates are more conservative. However, we reach exactly the same conclusions that he did; the Dunham Bay boat tail bullets were unstable at -10 F.

IV. Where Does Our Rule Come From?

Our Rule is based on correlating experimental data for moments of inertia, length, moment coefficients, and known twist and stability factor data. Here is how we got it.

The gyroscopic stability factor *s* at the **muzzle** is given by the formula **[McC99]** (see Technical Notes below)

$$s=\frac{8\pi}{\rho_{air}\,t^2\,d^5\,C_{M\alpha}}\frac{A^2}{B} \tag{1}$$

where *s* is the stability factor at the **muzzle** (dimensionless), ρ_{air} the density of air (lower case Greek rho), *t* the twist in **calibers/turn**, *d* the bullet diameter in **inches**, $C_{M\alpha}$ the overturning (pitching) moment coefficient (dimensionless), *A* the axial moment of inertia (units of weight$\times$ length^2), and *B* the transverse moment of inertia (units of weight$\times$ length^2). Note that *A*, *B*, ρ_{air}, and *d* must be in compatible units for *s* to be dimensionless.

The general equation for *s* is eq 1 multiplied by v_0^2/v^2, which gets larger as the bullet flies down range. However, *s* is also proportional to the spin rate, which gets smaller but much more slowly in comparison. Therefore *s* and the stability get larger going down range. Consequently, the stability factor at the *muzzle* is its lowest value and thus determines the necessary twist.

We analyzed 39 bullets from various BRL reports (mostly from **[B73]**), and found that we can approximately correlate the moments of inertia in the following dimensionless form (see Technical Notes):

$$\frac{B\,m\,d^2}{A^2} = 4.83\left(1+l^2\right) \qquad\qquad \text{non-dimensional} \qquad\qquad (2)$$

$$\frac{A^2}{B} = \frac{m\,d^2}{4.83\left(1+l^2\right)} \qquad\qquad (3)$$

We have also obtained a ***very*** approximate formula for $C_{M\alpha}$ at M=2.5 (2800 ft/sec) using data from the same sources:

$$C_{M\alpha} = C_{N\alpha}\left(cp - cg\right) \cong 2.85\left(0.6\,l - 0.4\,l\right) \cong 0.57\,l \qquad\qquad (4)$$

where *cp* is the center of pressure and *cg* is the center of gravity, both measured from the bullet's *base*. $C_{N\alpha}$ is the "normal force" coefficient.

Suppose we use the units grains for *m*, gr./in.³ for the density of air, and inches for *d*, then the Army Standard Metro air density (0.075126 lb/ft³) becomes 0.304330 gr./in.³. Then we get an equation for *s* by substitution of eq 3 and 4 in eq 1

$$s = \frac{30.0\,m}{t^2\,d^3\,l\left(1+l^2\right)} \qquad\qquad (5)$$

and in terms of twist

$$t^2 = \frac{30.0\,m}{s\,d^3\,l\left(1+l^2\right)} \qquad\qquad (6)$$

Notice that the density of the bullet doesn't appear here (as it does in the general Greenhill Rule), because it is contained in the moment of inertia approximation and is implicit in *m*.

The improved success of this Rule is partly because it is based on correlating modern experimental data. We also note that there are no shape factors included in our rule, whereas the moments of inertia, bullet weight, center of gravity, and overturning moment coefficient all depend on shape as well as length. Fortunately, their shape effects seem to partially cancel each other for cast, bronze, and solid core bullets. In particular, there is no clear difference between flat base and boat tail bullets. This makes the *l* in calibers the most important factor, as it is in the Greenhill Rule.

If *m* is in grams and *d* in cm (mm/10), the 30 in eq 7 is replaced by 30×252.9=7587, where 252.9 is a conversion factor between English and metric density units.

V. Effects of Velocity (Mach Number), Temperature, and Pressure

What about velocity effects on *s* and *t*?

The overturning moment coefficient $C_{M\alpha}$ is implicit in the denominator of eq A-C. Our correlation of it is based on a Mach number M= 2.5, or v=2800 ft/sec at standard conditions. However, $C_{M\alpha}$ *increases* as the velocity *decreases* and approaches the velocity of sound. This

increase in $C_{M\alpha}$ depends on the shape of the bullet. Near the sound velocity (1120 ft/sec), it can be 1.25 to 1.55 times larger than its value at M=2.5 (2800 ft/sec.) This means stability factors and twists are smaller below 2800 ft/sec and higher above. A **VERY** crude approximation for the velocity effect on the overturning moment is to multiply the calculated s by the **1/3** root of $v/2800$ and t by the **1/6** root of $v/2800$, where v is the velocity in ft/sec. Below the velocity of sound, use the velocity of sound value. This gives a correction factor

$$f_v = (v/2800)^{1/3} \tag{7}$$

for s and t^2 and the square root of f_v for t itself.

What about atmospheric temperature and pressure effects on s and t?

The Army Standard Metro conditions are temperature=59 deg Fahrenheit, pressure=750 mm Hg, and humidity=78%. The humidity effect is small enough to be ignored. At higher altitudes, the pressure and consequently the air density is less, so the bullet will be more stable and the formula is safer. At high temperatures, the air density is less so stability is increased. However, at low temperatures, the increased air density effect can be significant.

In recommending a safe stability factor above, we increased s from 1.75 to 2.0, based on the belief that no sane shooter will go out at temperatures lower than 15 or 20 below. However, the complete air density correction f_{TP} from standard conditions to actual atmospheric temperature and pressure comes from the Perfect Gas Law, and is

$$f_{TP} = \frac{\rho_{std}}{\rho_T} = \frac{(FT+460)}{(59+460)}\frac{P_{std}}{P_T} = \frac{(CT+273)}{(15+273)}\frac{P_{std}}{P_T} \tag{8}$$

where ρ_{std} is the air density at standard conditions, P the pressure, FT the Fahrenheit temperature, and CT the Celsius (centigrade) temperature. The individual ( ) are "absolute temperatures." Since the standard air density is in the denominator of eq 1, the correction factor **multiplies** the calculated s and t^2 and its square root multiplies t itself.

A general estimate of the effect of altitude on air density, using "conventional" T and P, is useful when actual measurements of T and P are not available. For Army Standard Metro, it is **[McC99]**

$$\frac{\rho_h}{\rho_{std}} = e^{(-3.158\times10^{-5}\times h)} \tag{9}$$

where h is the altitude in ft. The correction factor f_h that multiplies s and t^2 is

$$f_h = \frac{\rho_{std}}{\rho_h} = e^{(+3.158\times10^{-5}\times h)} \tag{10}$$

These correction factors f_v, f_{TP}, and f_h do not change t when calculating s, or s when calculating t.

By the way, the ICAO standard atmosphere is 59 deg F (15 deg Celsius), 760 mm Hg, and humidity 0%. The density of this atmosphere is only 2% larger than that of Army Standard Metro, small enough to be ignored in view of the approximations of eq 5.

VI. Technical Notes

a. Eq 1 is obtainable from many sources, in particular from eq 10.85 on page 230 in Robert L. McCoy's "Modern Exterior Ballistics" **[McC99]**, after changing to our notation, relating the spin to twist, and correcting misprints. (Unfortunately there are many publisher's misprints.) In his eq 10.85, the p in the denominator is actually the Greek letter ρ representing the air density, and his spin p in the numerator is related to the twist by $p = 2\pi v/(t\,d)$ (found on p192 below eq 9.31).

b. H. P. Hitchcock **[H52]** notes that good approximations for .30-.50 caliber ball and armor piercing bullets are: $A/md^2 = k_a^2 = 0.115$; $B/md^2 = k_t^2$ is proportional to $(1+0.944l^2)$; and $cg = 0.4l$ from the base. We note that k_t^2 for a cylinder is proportional to $(1.25+l^2)$, and for Greenhill's "football" shape to $(1+l^2)$.

c. $C_{M\alpha} = C_{N\alpha}(cp - cg)$ with cp and cg measured from the base. The empirical correction of the "slender body" approximation gives $C_{N\alpha} = 2.85$ (a good value at M=2.5=2800 ft/sec) and gives cp as the bullet volume in calibers[3]. This in turn is about $0.6l$ (a good value at M=2.5.) Therefore, with $cg=0.4l$, $C_{M\alpha} = 2.85(0.6l - 0.4l) = 0.57l$.

Good shooting!

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